CP violation in neutrino mixing matrix and leptogenesis

Yong Liu[1] and Utpal Sarkar[2]

[1] Lab of Numerical Study for Heliospheric Physics (LHP)
Chinese Academy of Sciences
P. O. Box 8701, Beijing 100080, P.R.China

[2] Physical Research Laboratory, Ahmedabad 380 009, India

Abstract

The CP violation required in leptogenesis may have different origin, but in an effective theory they all are related to the rephasing invariant CP violating measure in the mixing matrix of the leptonic sector. We point out that the maximum amount of CP violation in some models can be estimated with our present knowledge of the neutrino mixing angles, which can help us understand the CP violation in the generation of the lepton asymmetry of the universe. For example, the possibility of leptogenesis may be ruled out in some models from an knowledge of the effective neutrino mass matrix.

PACS number(s): 11.30.E, 14.60.P, 12.15.F

Recently, the Super-Kamiokande experiment has provided a strong evidence for non-zero masses and oscillations of neutrinos [1, 2]. Because it is one of the direct indications for new physics beyond the Standard Model, the announcement of the Super-Kamiokande result has brought up a turbulent shock in the research field of particle physics [3, 4]. Although the parameter space of the neutrino mass sector or the origin of the neutrino masses and mixing are yet to be known, these new experiments has narrowed down the allowed parameter space in the lepton sector.

It has now become an interesting exercise to understand the allowed parameter space in terms of different models or ansatz, as in the quark sector, with the hope to find an origin of the neutrino masses and mixing. There are different approches to the problem, namely, to postulate some ansatz for simplifying the problem and then check its consistency and predictability, or to consider some known ansatz and check if they are consistent. In analogy to the ansatze for the quark masses and mixing, one can assume a similar mass and mixing matrix for the neutrino sector. This would allow us to discuss the problem of mixing and CP violation in neutrino system naturally [5, 6, 7].

Another question of interest related to the neutrino mass is leptogenesis [8, 9]. It is now believed that the most promising mechanism for generating a baryon asymmetry of the universe is through lepton number violation. The scale of lepton number violation and the amount of CP violation tells us if it is possible to generate a lepton asymmetry of the universe at the lepton number violating scale, which then can get converted to a baryon asymmetry of the universe in the presence of the sphalerons. In general it is not possible to infer the amount of CP violation in the leptonic sector, since the CP phase is an independent parameter. However, given all the mixing angles it is possible to say what is the maximum amount of CP violation in any model in a rephasing invariant way. If this quantity vanishes, then one can infer that there is no CP violation in that model and hence leptogenesis will not be possible.

There has been several attempts to relate the various parameters in the quark sector with an aim to understand the origin of the quark masses and mixing. Different ansatz for the quark masses have been put forward to reduce the number of parameters. Some of these ansätze has been extended to the leptonic sector. In this article we shall study some of these models and estimate the maximum allowed CP violation and point out that from the study of the effective low energy mixing matrix one can rule out the possibility of leptogenesis in some cases.

We consider a three generation scenario with hierarchical Majorana masses, $m_{\nu_e} \ll$

Table 1: Present experimental constraints on neutrino masses and mixing

Solar Neutrino [10]	:	$\Delta m^2 \sim (0.8 - 2) \times 10^{-5} eV^2$
(Large angle MSW)		$\sin^2 2\theta \sim 1$
Solar Neutrino [10]	:	$\Delta m^2 \sim (0.5 - 1) \times 10^{-5} eV^2$
(Small angle MSW)		$\sin^2 2\theta \sim 10^{-2} - 10^{-3}$
Solar Neutrino [10]	:	$\Delta m^2 \sim (0.5 - 6) \times 10^{-10} eV^2$
(Vacuum oscillation)		$\sin^2 2\theta \sim 1$
Atmospheric Neutrino [1]	:	$\Delta m^2 \sim (0.5 - 6) \times 10^{-3} eV^2$
		$\sin^2 2\theta > 0.82$
Neutrinoless Double Beta Decay [11]	:	$m_{\nu_e} < 0.46 eV$
CHOOZ [12]	:	$\Delta m_{eX}^2 < 10^{-3} eV^2$
		(or $\sin^2 2\theta_{eX} < 0.2$)

$m_{\nu_\mu} \ll m_{\nu_\tau}$. The neutrino masses could originate from either see-saw mechanism [13] or through a triplet higgs field [9]. We assume that the neutrino mass matrix is such that it can explain the present experiments with the mass squared differences and mixing angles as given in table 1.

We shall further assume that the solar neutrino data is explained with $\nu_e - \nu_\mu$ oscillation, while the atmospheric data can be explained in terms of $\nu_\mu - \nu_\tau$ large mixing with a large mass splitting compared to the $\nu_e - \nu_\mu$ case [14]. Our result is valid when any two of the mixing angles are given, although we need not limit which two of the three mixing angles. For the solar neutrino problem we consider all the three possible solutions, namely the small angle MSW, the large angle MSW and the vacuum oscillation. The small-mixing solution causes the energy-spectrum distortion while the large-mixing solution causes the day-night flux difference, and the vacuum-oscillations cause seasonal variation of the ^7B_e solar neutrino flux [15]. Since we are interested in only the mixing angles, the large angle MSW solution and the vacuum oscillation solution would give us same result, *i.e.*, they both allow same amount of CP violation.

To understand the question of CP violation in the leptonic sector, we shall start with the neutrino mixing matrix V_ν, which we parametrize similar to the standard parametrization of the Cabibbo-Kaboyashi-Maskawa (CKM) matrix in the quark sec-

tor,

$$V_{KM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}. \tag{1}$$

where, the convention $s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij}$ (the "generation" labels $i, j = 1, 2, 3$) are used. δ_{13} and θ_{ij} are the CP phase and the mixing angles present in the mixing matrix present in the leptonic sector. We may work in the basis in which the charged lepton mass matrix is diagonal, in which case this is the matrix which diagonalises the neutrino mass matrix. With the real angles, θ_{12}, θ_{23} and θ_{13} can all be made to lie in the first quadrant. The phase δ_{13} lies in the range $0 < \delta_{13} < 2\pi$. In the following, we shall fix the three angles θ_{12}, θ_{23} and θ_{13} in the first quadrant.

Any rephasing of the neutrino fields can change the amount of CP violation in different sectors, but we can define a rephasing invariant quantity, similar to the Jarlskog invariant [16, 17] in the quark sector, given by,

$$J_{CP} = s_{12}s_{13}s_{23}c_{12}c_{13}^2 c_{23} \sin\delta_{13}. \tag{2}$$

This quantity is an measure of CP violation independent of the basis and phases. Neutrino masses could originate from any model which could have several sources of CP violation, but finally in terms of this effective theory all the sources of CP violation has to be related to this quantity J_{CP}.

In realistic models of neutrino masses one can integrate out the heavier fields (in the see-saw mechanism the right handed neutrinos and in the triplet higgs mechanism the heavy triplet scalars) and get an effective low energy scenario with three generation. Then diagonalising the charged lepton mass matrix one can obtain the neutrino mixing matrix and hence J_{CP}. No matter what are the sources of CP violation in the original model, if there is any CP violation to start with, then the final effective model will also violate CP and hence J_{CP} has to be non-vanishing. So, if we can predict J_{CP} in any model, we can infer about the existence of CP violation in the model. For example, to generate a lepton asymmetry of the universe one requires CP violation. If $J_{CP} = 0$ in any model, then it is not possible to generate a lepton asymmetry of the universe in that model, no matter how complicated the original model was.

Since the CP phase δ_{13} is an independent parameter, with our present knowledge it is not possible to predict J_{CP}. However, with our present knowledge of the mixing matrix V_ν we can compute the maximum permissible J_{CP}, which we call J_{CP}^{max}, by

choosing $\delta_{13} = \frac{\pi}{2}$. However, if we can predict δ_{13} starting from some ansatz or some other consideration, we can again estimate J_{CP}^{max}, although we may not estimate J_{CP}.

Let us now consider a few specific examples. Consider the bimaximal neutrino mixing matrix, in which $s_{13} = 0$. In this case, $J_{CP} = 0$, implying that there is no CP violation in the leptonic sector and hence leptogenesis is impossible in any model which produces exact bimaximal neutrino mixing matrix. Similarly, there are models with one sterile neutrino, where some texture neutrino mass matrix has been proposed [3, 18]. In these models one has to study a 4×4 mass matrix and hence there will be three J_{CP}. Although the weak mixing matrix will now be different from the neutrino mixing matrix, one can infer that the model is CP invariant if all the three J_{CP} vanishes. Because of the texture zeroes, all the J_{CP}^{max} vanishes in a few models (which will be discussed elsewhere), implying that although these textures are otherwise successful, they cannot come from any model which predicts non-vanishing lepton asymmetry of the universe.

One may then consider a deviation from the exact bimaximal neutrino mixing matrix and make $s_{13} \neq 0$, which will then have CP violation. Depending on the value of s_{13} the amount of CP violation will become uncertain. However, we can then use the CHOOZ data to give an upper bound on s_{13}, which will then allow us to predict J_{CP}^{max} for $\delta_{13} = \frac{\pi}{2}$.

¿From table 1, we can use the maximum allowed value of $\sin^2 2\theta_{23} \sim 1$. Then for the large angle MSW or vacuum oscillation solution of the solar neutrino problem we can again use $\sin^2 2\theta_{12} \sim 1$. For the third angle we can then use the CHOOZ result, $\sin^2 2\theta_{13} < 0.2$ to find an experimental upper bound on

$$J_{CP}^{max}(expt) < 0.056.$$

Similarly, for the small angle MSW solution of the solar neutrino problem we get,

$$J_{CP}^{max} < 0.005.$$

For this bound we considered $\sin^2 2\theta_{12} < .01$. In both these cases we assumed $\sin \delta_{13} \sim 1$. As can be seen from these expressions for J_{CP}^{max}, the amount of CP violation coming from the mixing matrix in the leptonic sector cannot be very large.

We shall next present a model of the weak CP violation in the quark sector [19, 20], which has a geometrical origin and has got several interesting observable predictions, which we would like to extend to the neutrino sector. Since the amount of CP violation is predicted in this model, we can estimate J_{CP}^{max} directly. In this model, the weak CP

phase δ_{13} has been related to the other three mixing angles θ_{12}, θ_{23} and θ_{13} by the relation,

$$\sin \delta_{13} = \frac{(1 + s_{12} + s_{23} + s_{13})\sqrt{1 - s_{12}^2 - s_{23}^2 - s_{13}^2 + 2s_{12}s_{23}s_{13}}}{(1 + s_{12})(1 + s_{23})(1 + s_{13})} \qquad (3)$$

The geometric interpretation comes from the fact that δ_{13} is the solid angle enclosed by $(\pi/2 - \theta_{12}), (\pi/2 - \theta_{23})$ and $(\pi/2 - \theta_{13})$ standing on a same point, or, the area to which the solid angle corresponding on a unit spherical surface. Hence, to make $(\pi/2 - \theta_{12}), (\pi/2 - \theta_{23})$ and $(\pi/2 - \theta_{13})$ be able to enclose a solid angle, the following relation must hold.

$$(\frac{\pi}{2} - \theta_{ij}) + (\frac{\pi}{2} - \theta_{jk}) \geq (\frac{\pi}{2} - \theta_{ki}) \qquad (i \neq j \neq k \neq i = 1, 2, 3. \quad \theta_{ij} = \theta_{ji}) \qquad (4)$$

With the constraints Eq.(4) and Eq.(3) we shall now study the predictions of the CP violation in this scenario.

The atmospheric neutrino problem requires,

$$\theta_{\mu\tau} \approx \pi/4. \qquad (5)$$

This will give restriction on the mixing angle between ν_e and ν_τ. From Eq.(4), we have

$$|(\frac{\pi}{2} - \theta_{e\mu}) - (\frac{\pi}{2} - \theta_{\mu\tau})| \leq (\frac{\pi}{2} - \theta_{e\tau}) \leq Min(\pi/2, \ (\frac{\pi}{2} - \theta_{e\mu}) + (\frac{\pi}{2} - \theta_{\mu\tau})) \qquad (6)$$

Note that, we can read off the mixing angles from table 1, which implies for the small and large angle MSW solutions, to be

$$\theta_{e\mu} \sim 0.045 \quad \text{or} \quad \pi/2 - 0.045$$

and

$$\theta_{e\mu} \sim 0.7 \quad \text{or} \quad \pi/2 - 0.7$$

respectively. Considering Eq.(5), then we obtain

$$0 \leq \theta_{e\tau} \leq \pi/4 + 0.045 \qquad (7)$$

or

$$\pi/4 - 0.045 \leq \theta_{e\tau} \leq \pi/4 + 0.045 \qquad (8)$$

for the case of small-mixing solution. And

$$0 \leq \theta_{e\tau} \leq \pi/4 + 0.7 \qquad (9)$$

or

$$\pi/4 - 0.7 \leq \theta_{e\tau} \leq \pi/4 + 0.7 \tag{10}$$

for the case of large-mixing solution. Although eq.(7-10) seems to be the new constraints in this scenario, they are irrelevant. Considering the CHOOZ data we can easily see that the region allowed in this scenario is just the region allowed by CHOOZ,

$$0 \leq \sin \theta_{e\tau} \leq .23. \tag{11}$$

Substituting eqn (3) into eqn (2), we obtain J_{CP} as a function of $\theta_{e\tau}$ and can draw the curve with J_{CP} versus $\theta_{e\tau}$. The results are shown in fig. 1. ¿From the figure we can put a limit on the amount of CP violation, from the limit on $\theta_{e\tau}$ to be,

$$J_{CP} < 0.0015. \tag{12}$$

However, there is another direct way to give bound on the amount of CP violation in this scenario. For this we assume that J_{CP} corresponds to the largest value of δ_{13} (which we can verify from the graph).

Using eq.(3) we can get an upper bound on the CP phase in this parametrization to be $\sin \delta_{13} \sim .13$ for the large angle solution of the solar neutrino problem, so that $\sin^2 2\theta_{e\mu} \sim 1$ and $\sin^2 2\theta_{\mu\tau} \sim 1$ and maximum allowed value for the third angle to be given by CHOOZ, $\sin^2 2\theta_{e\tau} < .2$. These values will then give a maximum allowed value for the rephasing invariant CP violating qantity,

$$J_{CP}^{max} < 0.0175. \tag{13}$$

On the other hand for the small angle MSW solution the CP phase is predicted to be larger, $\delta_{13} \sim 0.61$ and the rephasing invariant CP violating quantity becomes becomes

$$J_{CP}^{max} < 0.0034. \tag{14}$$

Again we obtain a maximum measure of CP violation using the largest value of $\sin^2 2\theta_{12} \sim 0.01$. In both the cases the amount of maximum CP violation is much lower than the experimental bounds.

To summarise, we have shown that it is possible to estimate the maximum allowed value of the rephasing invariant measure of CP violation in a given model if we know all the three angles. Since the CP phase is an independent parameter, one can assume a maximum value of unity for this quantity to calculate the rephasing invariant CP violating parameter. In exactly bimaximal mixing model and in some models of textured

neutrino mass matrix, this measure J_{CP}^{max} vanishes implying no CP violation, whereas in another geometric model of weak CP phase there is a large suppression. Any ansatz of the neutrino mixing matrix can, in general, suppress this quantity, which in turn will suppress the amount of CP violation in that model, whose direct effect will be on the amount of lepton asymmetry of the universe. In models with $J_{CP}^{max} = 0$, it is not possible to have lepton asymmetry of the universe.

References

[1] Super-Kamiokande Collaboration : Y. Fukuda *et al*, Phys. Rev. Lett. **81**, 1562 (1998); hep-ex/9805006; Phys. Lett. **B 433**, 9 (1998); T. Kajita, hep-ex/9810001.

[2] Super-Kamiokande Collaboration: Y. Fukuda *et al*, Phys. Rev. Lett. **81**, 1158 (1998); Talk by Y. Suzuki at Neutrino'98, Takayama, Japan (1998).

[3] V. Barger, T.J. Weiler and K. Whisnant, Phys. Lett. **B 427** (1998) 97; V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, hep-ph/9806328; S.C. Gibbons, R.N. Mohapatra, S. Nandi and A. Raychaudhuri, Phys. Lett. **B 430** (1998) ; R.N. Mohapatra and S. Nusinov, hep-ph/9808301; H. Fritzsch and Z. Xing, hep-ph/9808272; G. Cleaver, M. Cvetic, J.R. Espinosa, L Everett and P. Langacker, Phys. Rev. **D 57** (1998) 2701; Subhendra Mohanty, D.P. Roy, Utpal Sarkar, Phys. Lett. **B445** (1998) 185.

[4] V. Barger, S. Pakvasa, T.J. Weiler and K. Whisnant, hep-ph/9806387; G. Alterelli and F. Feruglio, hep-ph/9807353; J. .Ellis et al, hep-ph/9808301; H. Georgi and S.L. Glashow, hep-ph/9808293; R. Barbieri, L.J. Hall and A. Strumia hep-ph/9808333; Y. Grossman, Y.Nir and Y. Shadmi, hep-ph/9808355; N. Gaur, A. Ghosal, E. Ma and P. Roy, report no. hep-ph/9806272.Ernest Ma, D.P. Roy and Utpal Sarkar, Phys. Lett. **B444** (1998) 391; U. Sarkar, Phys. Rev. **D 59** (1999) 037302; **D59** (1999) 031301; A. Joshipura, hep-ph/9808261; A. Joshipura and S. Vempati, hep-ph/9808232.

[5] K. R. Schubert, hep-ph/9902215.

[6] A. M. Gago, V. Pleitez and R. Zukanovich Funchal, hep-ph/9810505.

[7] V. Barger, K. Whisnant and R. J. N. Phillips, Phys. Rev. Lett. **45**, 2084 (1980).

[8] M. Fukugita and T. Yanagida, Phys. Lett. **B 174** (1986) 45; P. Langacker, R. Peccei and T. Yanagida, Mod. Phys. Lett. **A 1**, 541 (1986); M.A. Luty, Phys. Rev. **D 45** (1992) 445; A. Acker, H. Kikuchi, E. Ma and U. Sarkar, Phys. Rev. **D 48** (1993) 5006; T. Gherghetta and G. Jungman, Phys.Rev. **D 48** (1993) 1546; M. Flanz, E.A. Paschos and U. Sarkar, Phys. Lett. **B 345** (1995) 248; W. Buchmüller and M. Plümacher, Phys. Lett. **B 389** (1996) 73; Phys. Lett. **431** (1998) 354; L. Covi, E. Roulet and F. Vissani, Phys. Lett. **B 384** (1996) 169; M. Flanz, E.A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. **B 389** (1996) 693; A. Pilaftsis, Phys. Rev. **D 56** (1997) 5431; hep-ph/9810211.

[9] E. Ma and U. Sarkar, Phys. Rev. Lett. **80** (1998) 5716; E. Ma, M. Raidal and U. Sarkar, hep-ph/9811240; hep-ph/9808484; hep-ph/9901406; E. Ma, S. Sarkar and U. Sarkar, hep-ph/9812276; D. Delepine and U. Sarkar, hep-ph/9811479.

[10] J. Bahcall and H. Bethe, Phys. Rev. Lett. **65**, 2233(1990) and Phys. Rev. D**44**, 2962(1991). N. Hata and P. Langacker, Phys. Rev. D**56**, 6107(1997); J.N. Bahcall, P.J. Krastev and A.Yu. Smirnov, hep-ph/9807216.

[11] H.V. Klapdor-Kleingrothaus, in Proc Lepton and Baryon number violatoin, Trento, April 1998; M. Günther et al, Phys. Rev. **D 55** (1997) 54; L. Baudis et al, Phys. Lett. **B 407** (1997) 219.

[12] CHOOZ Collaboration : M. Appollonio et al, hep-ex/9711002.

[13] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, ed. by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam); T. Yanagida, in *Proc of the Workshop on Unified Theories and Baryon Number in the Universe*, Tsukuba, Japan, edited by A. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba) 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44** (1980) 912.

[14] R. Barbieri et al., hep-ph/9807235. E. Malkawi, hep-ph/9810542.

[15] Particle Data Group, The Eur. Phys. Jour. **C 3** (1998) 310.

[16] *CP Violation* Ed. C. Jarlskog, World Scientific Publishing Co. Pte. Ltd 1989.

[17] E. A. Paschos and U. Turke, Phys. Rep.**4**, 145(1989).

[18] S. Nandi and U. Sarkar, Phys. Rev. Lett. **56** (1986) 564; A.S. Joshipura and U. Sarkar, Phys. Rev. Lett. **57** (1986) 33; A. Masiero, D.V. Nanopoulos and A.I. Sanda, Phys. Rev. Lett. **57** (1986) 663; A. Smirnov, talk given at the 28th International Conference on High Energy Physics (1996), Warsaw, Poland (hep-ph/9611465);

[19] Chen Jing-Ling, Mo-Lin Ge, Xue-Qian Li and Yong Liu, hep-ph/9809527 (To appear in Eur. Phys. Jour. C).
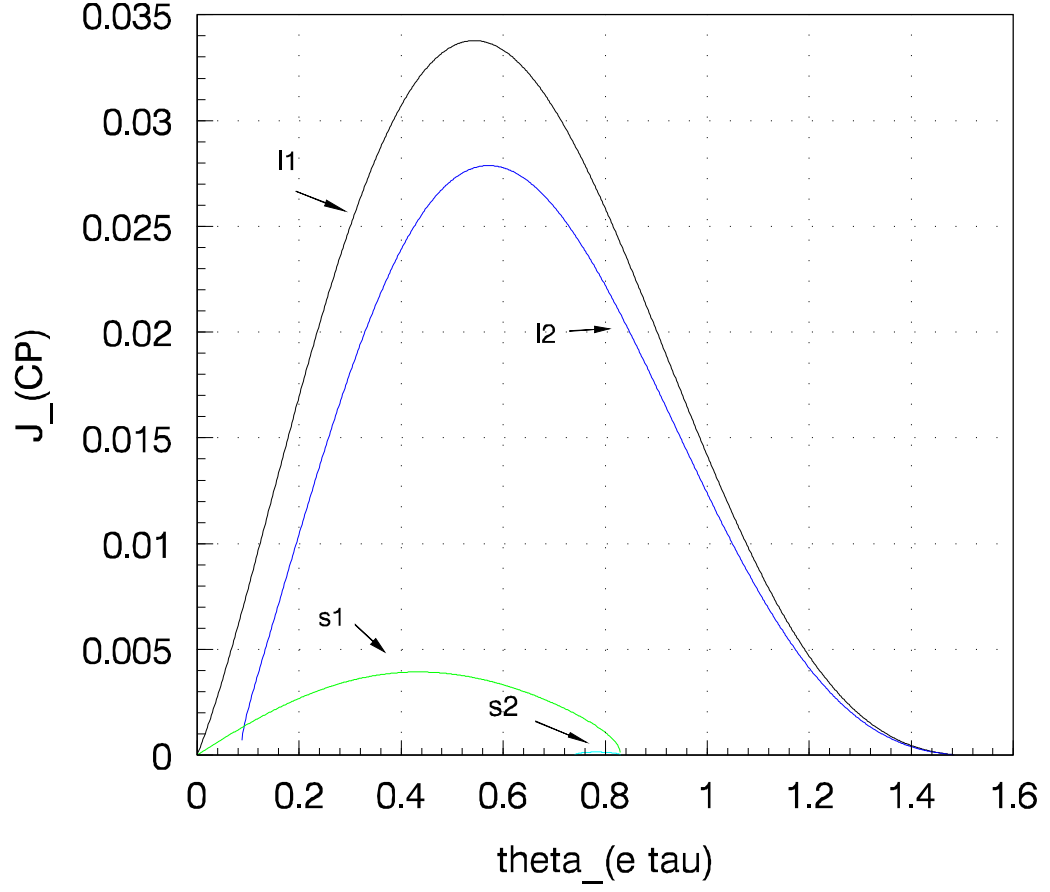
[20] Yong Liu, hep-ph/9811508.

Figure 1: J_{CP} versus $\theta_{e\tau}$. Where, $\theta_{\mu\tau} = \pi/4$. The curves s1, s2, l1 and l2 corresponds to the cases of $\theta_{e\mu} = 0.045$, $(\pi/2 - 0.045)$, 0.443 and $(\pi/2 - 0.443)$ respectively.